N13/5/MATHL/HP2/ENG/TZ0/XX/M

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

# MARKSCHEME

## November 2013

## MATHEMATICS

## Higher Level

## Paper 2

20 pages

## Instructions to Examiners

### Abbreviations

***M*** Marks awarded for attempting to use a correct **Method**; working must be seen.

***(M)*** Marks awarded for **Method**; may be implied by **correct** subsequent working.

***A*** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

***(A)*** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

***R*** Marks awarded for clear **Reasoning**.

***N*** Marks awarded for **correct** answers if **no** working shown.

***AG*** Answer given in the question and so no marks are awarded.

### Using the markscheme

**1 General**

Mark according to Scoris instructions and the document **"Mathematics HL: Guidance for e-marking November 2013"**. It is essential that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the 'must be seen' marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by Scoris.

**2 Method and Answer/Accuracy marks**

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, for example, ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (for example, substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

**3 *N* marks**

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

**4 Implied marks**

*Implied marks appear in **brackets**, for example, **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

**5 Follow through marks**

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

**6 Mis-read**

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

**7 Discretionary marks *(d)***

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

**8 Alternative methods**

*Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.*

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

**9 Alternative forms**

*Unless the question specifies otherwise,* ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

**Example**: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

**10 Accuracy of Answers**

*Candidates should* ***NO LONGER*** *be penalized for an accuracy error* ***(AP)***.

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for* ***FT***.

**11 Crossed out work**

*If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.*

**12 Calculators**

*A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.*

**Calculator notation**
The Mathematics HL guide says:

*Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

**13 More than one solution**

*Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.*

## 14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

## SECTION A

**1.** $\boldsymbol{AX} = \boldsymbol{B}$

**EITHER**

$\Rightarrow \boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$ *(M1)*

**OR**

attempting row reduction:

*eg* $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -1 & -6 \\ 0 & -2 & 0 & -1 \end{array}\right)$ *(M1)*

**THEN**

$\Rightarrow \boldsymbol{X} = \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \\ 5 \end{pmatrix}$ *A1A1A1*

***Total [4 marks]***

**2.** (a) **METHOD 1**

$34 = a + 3d$ and $76 = a + 9d$ *(M1)*
$d = 7$ *A1*
$a = 13$ *A1*

**METHOD 2**

$76 = 34 + 6d$ *(M1)*
$d = 7$ *A1*
$34 = a + 21$
$a = 13$ *A1*

***[3 marks]***

(b) $\frac{n}{2}\left(26 + 7\left(n - 1\right)\right) > 5000$ *(M1)(A1)*
$n > 36.463...$ *(A1)*

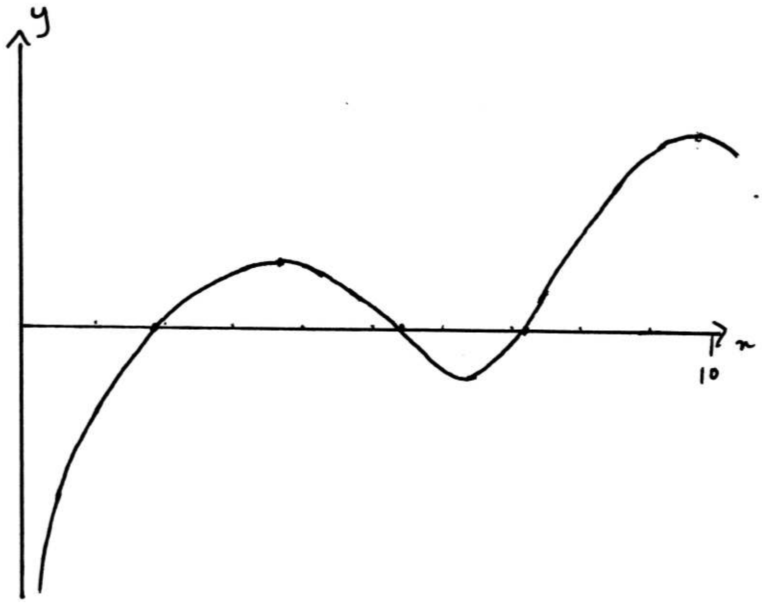
| | |
|---|---|
| **Note:** | Award ***M1A1A1*** for using either an equation, a graphical approach or a numerical approach. |

$n = 37$ *A1* *N3*

***[4 marks]***

***Total [7 marks]***

**3.** (a)




A correct graph shape for $0 < x \leq 10$. *A1*

maxima $(3.78, 0.882)$ and $(9.70, 1.89)$ *A1*

minimum $(6.22, -0.885)$ *A1*

$x$-axis intercepts $(1.97, 0)$, $(5.24, 0)$ and $(7.11, 0)$ *A2*

**Note:** Award ***A1*** if two $x$-axis intercepts are correct.

***[5 marks]***

(b) $0 < x \leq 1.97$ *A1*

$5.24 \leq x \leq 7.11$ *A1*

***[2 marks]***

***Total [7 marks]***

**4.** $\mathrm{P}\left(Z < \frac{780-\mu}{\sigma}\right) = 0.92$ and $\mathrm{P}\left(Z < \frac{755-\mu}{\sigma}\right) = 0.12$ ***(M1)***

use of inverse normal ***(M1)***

$\Rightarrow \frac{780-\mu}{\sigma} = 1.405\ldots$ and $\frac{755-\mu}{\sigma} = -1.174\ldots$ ***(A1)***

solving simultaneously ***(M1)***

| |
|---|
| **Note:** Award ***M1*** for attempting to solve an incorrect pair of equations *eg,* inverse normal not used. |

$\mu = 766.385$
$\sigma = 9.6897$
$\mu = 12$ hrs 46 mins $(= 766$ mins$)$ ***A1***
$\sigma = 10$ mins ***A1***

***Total [6 marks]***

**5.** (a) $\mathrm{P}(F) = \left(\frac{1}{7} \times \frac{7}{9}\right) + \left(\frac{6}{7} \times \frac{4}{9}\right)$ ***(M1)(A1)***

| |
|---|
| **Note:** Award ***M1*** for the sum of two products. |

$= \frac{31}{63}$ $(= 0.4920\ldots)$ ***A1***

***[3 marks]***

(b) Use of $\mathrm{P}(S \mid F) = \frac{\mathrm{P}(S \cap F)}{\mathrm{P}(F)}$ to obtain $\mathrm{P}(S \mid F) = \frac{\frac{1}{7} \times \frac{7}{9}}{\frac{31}{63}}$. ***M1***

| |
|---|
| **Note:** Award ***M1*** only if the numerator results from the product of two probabilities. |

$= \frac{7}{31}$ $(= 0.2258\ldots)$ ***A1***

***[2 marks]***

***Total [5 marks]***

**6.** (a) $\frac{a+\mathrm{i}}{a-\mathrm{i}} \times \frac{a+\mathrm{i}}{a+\mathrm{i}}$ ***M1***

$= \frac{a^2-1+2a\mathrm{i}}{a^2+1} \left( = \frac{a^2-1}{a^2+1} + \frac{2a}{a^2+1}\mathrm{i} \right)$ ***A1***

(i) $z$ is real when $a = 0$ ***A1***

(ii) $z$ is purely imaginary when $a = \pm 1$ ***A1***

| |
|---|
| **Note:** Award ***M1A0A1A0*** for $\frac{a^2-1+2a\mathrm{i}}{a^2-1} \left( = 1 + \frac{2a}{a^2-1}\mathrm{i} \right)$ leading to $a = 0$ in (i). |

***[4 marks]***

(b) **METHOD 1**

attempting to find either $|z|$ or $|z|^2$ by expanding and simplifying

*eg* $|z|^2 = \frac{(a^2-1)^2 + 4a^2}{(a^2+1)^2} = \frac{a^4+2a^2+1}{(a^2+1)^2}$ ***M1***

$= \frac{(a^2+1)^2}{(a^2+1)^2}$

$|z|^2 = 1 \Rightarrow |z| = 1$ ***A1***

**METHOD 2**

$|z| = \frac{|a+\mathrm{i}|}{|a-\mathrm{i}|}$ ***M1***

$|z| = \frac{\sqrt{a^2+1}}{\sqrt{a^2+1}} \Rightarrow |z| = 1$ ***A1***

***[2 marks]***

***Total [6 marks]***

**7.** (a) attempting to form $(3\cos\theta+6)(\cos\theta-2)+7(1+\sin\theta)=0$ ***M1***

$3\cos^2\theta-12+7\sin\theta+7=0$ ***A1***

$3\left(1-\sin^2\theta\right)+7\sin\theta-5=0$ ***M1***

$3\sin^2\theta-7\sin\theta+2=0$ ***AG***

***[3 marks]***

(b) attempting to solve algebraically (including substitution) or graphically for $\sin\theta$ ***(M1)***

$\sin\theta=\frac{1}{3}$ ***(A1)***

$\theta=0.340\quad(=19.5^\circ)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**8.** (a) $A=\frac{1}{2}\times10^2\times\theta-\frac{1}{2}\times10^2\times\sin\theta$ ***M1A1***

| **Note:** Award ***M1*** for use of area of segment = area of sector − area of triangle. |
|---|

$=50\theta-50\sin\theta$ ***AG***

***[2 marks]***

(b) **METHOD 1**

unshaded area $=\frac{\pi\times10^2}{2}-50(\theta-\sin\theta)$

(or equivalent *eg* $50\pi-50\theta+50\sin\theta$) ***(M1)***

$50\theta-50\sin\theta=\frac{1}{2}(50\pi-50\theta+50\sin\theta)$ ***(A1)***

$3\theta-3\sin\theta-\pi=0$

$\Rightarrow\theta=1.969$ (rad) ***A1***

**METHOD 2**

$50\theta-50\sin\theta=\frac{1}{3}\left(\frac{\pi\times10^2}{2}\right)$ ***(M1)(A1)***

$3\theta-3\sin\theta-\pi=0$

$\Rightarrow\theta=1.969$ (rad) ***A1***

***[3 marks]***

***Total [5 marks]***

**9.** (a) **METHOD 1**

for P on $L_1$, $\overrightarrow{\text{OP}} = \begin{pmatrix} -5-\lambda \\ -3+2\lambda \\ 2+2\lambda \end{pmatrix}$

require $\begin{pmatrix} -5-\lambda \\ -3+2\lambda \\ 2+2\lambda \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = 0$ ***M1***

$5+\lambda-6+4\lambda+4+4\lambda=0$ (or equivalent) ***A1***

$\lambda = -\frac{1}{3}$ ***A1***

$\therefore \overrightarrow{\text{OP}} = \begin{pmatrix} -\frac{14}{3} \\ -\frac{11}{3} \\ \frac{4}{3} \end{pmatrix}$ ***A1***

$L_2 : \boldsymbol{r} = \mu \begin{pmatrix} -14 \\ -11 \\ 4 \end{pmatrix}$ ***A1***

**Note:** Do not award the final ***A1*** if $\boldsymbol{r} =$ is not seen.

***[5 marks]***

**METHOD 2**

Calculating either $\left|\overrightarrow{\text{OP}}\right|$ or $\left|\overrightarrow{\text{OP}}\right|^2$ *eg*

$\left|\overrightarrow{\text{OP}}\right| = \sqrt{(-5-\lambda)^2 + (-3+2\lambda)^2 + (2+2\lambda)^2}$

$= \sqrt{9\lambda^2 + 6\lambda + 38}$ ***A1***

Solving either $\frac{\text{d}}{\text{d}\lambda}\left(\left|\overrightarrow{\text{OP}}\right|\right) = 0$ or $\frac{\text{d}}{\text{d}\lambda}\left(\left|\overrightarrow{\text{OP}}\right|^2\right) = 0$ for $\lambda$. ***M1***

$\lambda = -\frac{1}{3}$ ***A1***

$\overrightarrow{\text{OP}} = \begin{pmatrix} -\frac{14}{3} \\ -\frac{11}{3} \\ \frac{4}{3} \end{pmatrix}$ ***A1***

$L_2 : \boldsymbol{r} = \mu \begin{pmatrix} -14 \\ -11 \\ 4 \end{pmatrix}$ ***A1***

**Note:** Do not award the final ***A1*** if $\boldsymbol{r} =$ is not seen. ***[5 marks]***

*continued…*

*Question 9 continued*

(b) **METHOD 1**

$\left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{\left(-\frac{14}{3}\right)^2 + \left(-\frac{11}{3}\right)^2 + \left(\frac{4}{3}\right)^2}$ ***(M1)***

$= 6.08 \quad \left(= \sqrt{37}\right)$ ***A1***

**METHOD 2**

$\text{shortest distance} = \frac{\left|\begin{pmatrix}-1\\2\\2\end{pmatrix} \times \begin{pmatrix}-5\\-3\\2\end{pmatrix}\right|}{\left|\begin{pmatrix}-1\\2\\2\end{pmatrix}\right|}$ ***(M1)***

$= \frac{|10\boldsymbol{i} + 8\boldsymbol{j} + 13\boldsymbol{k}|}{\sqrt{1+4+4}}$

$= 6.08 \left(= \sqrt{37}\right)$ ***A1***

***[2 marks]***

***Total [7 marks]***

**10. EITHER**

$\frac{dx}{du} = 2\sec^2 u$ ***A1***

$\int \frac{2\sec^2 u \, du}{4\tan^2 u\sqrt{4+4\tan^2 u}}$ ***(M1)***

$= \int \frac{2\sec^2 u \, du}{4\tan^2 u \times 2\sec u}$ $\left(= \int \frac{du}{4\sin^2 u\sqrt{\tan^2 u+1}} \text{ or } = \int \frac{2\sec^2 u \, du}{4\tan^2 u\sqrt{4\sec^2 u}}\right)$ ***A1***

**OR**

$u = \arctan \frac{x}{2}$

$\frac{du}{dx} = \frac{2}{x^2+4}$ ***A1***

$\int \frac{\sqrt{4\tan^2 u + 4} \, du}{2 \times 4\tan^2 u}$ ***(M1)***

$\int \frac{2\sec u \, du}{2 \times 4\tan^2 u}$ ***A1***

**THEN**

$= \frac{1}{4}\int \frac{\sec u \, du}{\tan^2 u}$

$= \frac{1}{4}\int \operatorname{cosec} u \cot u \, du$ $\left(= \frac{1}{4}\int \frac{\cos u}{\sin^2 u} du\right)$ ***A1***

$= -\frac{1}{4}\operatorname{cosec} u \ (+C)$ $\left(= -\frac{1}{4\sin u} \ (+C)\right)$ ***A1***

use of either $\tan u = \frac{x}{2}$ or an appropriate trigonometric identity ***M1***

either $\sin u = \frac{x}{\sqrt{x^2+4}}$ or $\operatorname{cosec} u = \frac{\sqrt{x^2+4}}{x}$ (or equivalent) ***A1***

$= \frac{-\sqrt{x^2+4}}{4x} (+C)$ ***AG***

***Total [7 marks]***

**SECTION B**

**11.** (a) (i) $X \sim \text{Po}(0.6)$

$\text{P}(X = 0) = 0.549 \ \left(= \text{e}^{-0.6}\right)$ ***A1***

(ii) $\text{P}(X \geq 3) = 1 - \text{P}(X \leq 2)$ ***(M1)(A1)***

$= 1 - \left(\text{e}^{-0.6} + \text{e}^{-0.6} \times 0.6 + \text{e}^{-0.6} \times \dfrac{0.6^2}{2}\right)$

$= 0.0231$ ***A1***

(iii) $Y \sim \text{Po}(2.4)$ ***(M1)***

$\text{P}(Y \leq 5) = 0.964$ ***A1***

(iv) $Z \sim \text{B}(12,\ 0.451...)$ ***(M1)(A1)***

**Note:** Award ***M1*** for recognising binomial and ***A1*** for using correct parameters.

$\text{P}(Z = 4) = 0.169$ ***A1***

***[9 marks]***

(b) (i) $k\int_1^3 \ln x \, \text{d}x = 1$ ***(M1)***

$(k \times 1.2958... = 1)$

$k = 0.771702$ ***A1***

(ii) $\text{E}(X) = \int_1^3 kx \ln x \, \text{d}x$ ***(A1)***

attempting to evaluate their integral ***(M1)***

$= 2.27$ ***A1***

(iii) $x = 3$ ***A1***

(iv) $\int_1^m k \ln x \, \text{d}x = 0.5$ ***(M1)***

$k[x \ln x - x]_1^m = 0.5$

attempting to solve for $m$ ***(M1)***

$m = 2.34$ ***A1***

***[9 marks]***

***Total [18 marks]***

**12.** (a) (i) **METHOD 1**

$v = \int 3\cos\frac{t}{4}\,\mathrm{d}t$ ***M1***

$= 12\sin\frac{t}{4} + c$ ***A1***

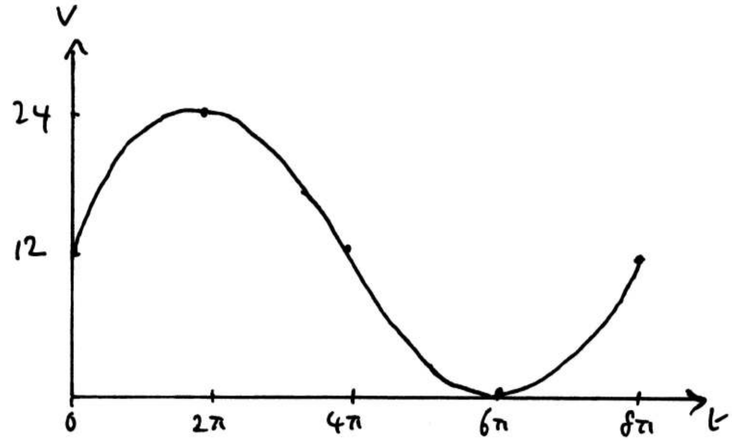
$t = 0,\ v = 12 \Rightarrow v = 12\sin\frac{t}{4} + 12$ ***A1***

**METHOD 2**

$v - 12 = \int_0^t 3\cos\frac{t}{4}\,\mathrm{d}t$ ***M1A1***

$v = 12\sin\frac{t}{4} + 12$ ***A1***

(ii)




***A1A1A1***

**Note:** Award ***A1*** for shape and domain $0 \le t \le 8\pi$.
Award ***A1*** for $(0, 12)$ and $(6\pi, 0)$ $\left((18.8, 0)\right)$.
Award ***A1*** for $(2\pi, 24)$ $\left((6.28, 24)\right)$.

(iii) **METHOD 1**

$\int_0^{6\pi}\left(12\sin\frac{t}{4} + 12\right)\mathrm{d}t$ ***M1***

$= 274\ (\text{m})\ \left(= 72\pi + 48\ (\text{m})\right)$ ***A1***

**METHOD 2**

$s = \int 12\sin\frac{t}{4} + 12\ \mathrm{d}t$

$= -48\cos\frac{t}{4} + 12t + c$ ***M1***

When $t = 0$, $s = 0$ and so $c = 48$.

When $t = 6\pi$, $s = 274\ (\text{m})\ \left(= 72\pi + 48\ (\text{m})\right)$. ***A1***

***[8 marks]***

*continued …*

*Question 12 continued*

(b) (i) **METHOD 1**

$\frac{\mathrm{d}v}{\mathrm{d}t} = -(v^2 + 4)$ ***(A1)***

$\int \frac{\mathrm{d}v}{v^2 + 4} = -\int \mathrm{d}t$ ***M1***

$\frac{1}{2}\arctan\left(\frac{v}{2}\right) = -t + c$ ***A1***

**EITHER**

$t = 0,\ v = 2 \Rightarrow c = \frac{\pi}{8}$ ***M1***

$\arctan\left(\frac{v}{2}\right) = \frac{\pi}{4} - 2t$ ***A1***

**OR**

$v = 2\tan(2c - 2t)$ ***A1***

$t = 0,\ v = 2 \Rightarrow \tan 2c = 1$ and so $c = \frac{\pi}{8}$ ***M1***

**THEN**

$v = 2\tan\left(\frac{\pi}{4} - 2t\right)$ ***A1***

$v = 2\tan\left(\frac{\pi - 8t}{4}\right)$ ***AG***

**METHOD 2**

$\frac{dv}{dt} = -4\sec^2\left(\frac{\pi - 8t}{4}\right)$ ***M1A1***

Substituting $v = 2\tan\left(\frac{\pi - 8t}{4}\right)$ into $\frac{dv}{dt} = -(v^2 + 4)$:

$\frac{dv}{dt} = -\left(4\tan^2\left(\frac{\pi - 8t}{4}\right) + 4\right)$ ***M1***

$= -4\left(\tan^2\left(\frac{\pi - 8t}{4}\right) + 1\right)$ ***(A1)***

$= -4\sec^2\left(\frac{\pi - 8t}{4}\right)$ ***A1***

Verifying that $v = 2$ when $t = 0$. ***A1***

*continued ...*

(ii) **METHOD 1**

$v\dfrac{\mathrm{d}v}{\mathrm{d}s} = -\left(v^2+4\right)$ ***A1***

$\Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}s} = -\dfrac{\left(v^2+4\right)}{v}$ ***AG***

**METHOD 2**

$\dfrac{\mathrm{d}v}{\mathrm{d}s} = \dfrac{\mathrm{d}v}{\mathrm{d}t} \times \dfrac{\mathrm{d}t}{\mathrm{d}s}$ ***A1***

$\Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}s} = -\dfrac{\left(v^2+4\right)}{v}$ ***AG***

(iii) **METHOD 1**

When $v=0$, $t=\dfrac{\pi}{8}$ $(t=0.392...)$. ***(M1)A1***

$s=\int_0^{\frac{\pi}{8}} 2\tan\left(\dfrac{\pi-8t}{4}\right)\mathrm{d}t$ ***(M1)***

$s=0.347$ (m) $\left(s=\dfrac{1}{2}\ln 2 \text{ (m)}\right)$ ***A2***

**METHOD 2**

$\int \dfrac{v}{4+v^2}\mathrm{d}v = -\int \mathrm{d}s$ ***M1***

**EITHER**

$\dfrac{1}{2}\ln\left(v^2+4\right) = -s+c$ (or equivalent) ***A1***

$v=2,\ s=0 \Rightarrow c=\dfrac{1}{2}\ln 8$ ***M1***

$s=-\dfrac{1}{2}\ln\left(v^2+4\right)+\dfrac{1}{2}\ln 8$ $\left(s=\dfrac{1}{2}\ln\left(\dfrac{8}{v^2+4}\right)\right)$ ***(A1)***

$v=0 \Rightarrow s=\dfrac{1}{2}\ln 2$ (m) $\left(s=0.347 \text{ (m)}\right)$ ***A1***

**OR**

$-\int_2^0 \dfrac{v}{4+v^2}\mathrm{d}v = s$ (or equivalent) ***M1A1***

| **Note:** Award ***M1*** for setting up a definite integral and award ***A1*** for stating correct limits. |
|---|

$s=0.347$ (m) $\left(s=\dfrac{1}{2}\ln 2 \text{ (m)}\right)$ ***A2***

***[12 marks]***

***Total [20 marks]***

**13.** (a) (i) either counterexample or sketch or
recognising that $y = k$ $(k > 1)$ intersects the graph of $y = f(x)$ twice ***M1***
function is not 1–1 (does not obey horizontal line test) ***R1***
so $f^{-1}$ does not exist ***AG***

(ii) $f'(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$ ***(A1)***

$f'(\ln 3) = \frac{4}{3}$ $(= 1.33)$ ***(A1)***

$m = -\frac{3}{4}$ ***M1***

$f(\ln 3) = \frac{5}{3}$ $(= 1.67)$ ***A1***

**EITHER**

$\dfrac{y - \frac{5}{3}}{x - \ln 3} = -\frac{3}{4}$ ***M1***

$4y - \frac{20}{3} = -3x + 3\ln 3$ ***A1***

**OR**

$\frac{5}{3} = -\frac{3}{4}\ln 3 + c$ ***M1***

$c = \frac{5}{3} + \frac{3}{4}\ln 3$

$y = -\frac{3}{4}x + \frac{5}{3} + \frac{3}{4}\ln 3$ ***A1***

$12y = -9x + 20 + 9\ln 3$

**THEN**

$9x + 12y - 9\ln 3 - 20 = 0$ ***AG***

(iii) The tangent at $\left(a, f(a)\right)$ has equation $y - f(a) = f'(a)(x - a)$. ***(M1)***

$f'(a) = \frac{f(a)}{a}$ (or equivalent) ***(A1)***

$e^{a} - e^{-a} = \frac{e^{a} + e^{-a}}{a}$ (or equivalent) ***A1***

attempting to solve for $a$ ***(M1)***
$a = \pm 1.20$ ***A1A1***

***[14 marks]***

*continued …*

*Question 13 continued*

(b) (i) $2y = e^x + e^{-x}$

$e^{2x} - 2ye^x + 1 = 0$ ***M1A1***

> **Note:** Award ***M1*** for either attempting to rearrange or interchanging *x* and *y*.

$e^x = \dfrac{2y \pm \sqrt{4y^2 - 4}}{2}$ ***A1***

$e^x = y \pm \sqrt{y^2 - 1}$

$x = \ln\left(y \pm \sqrt{y^2 - 1}\right)$ ***A1***

$f^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$ ***A1***

> **Note:** Award ***A1*** for correct notation and for stating the positive "branch".

(ii) $V = \pi \int_1^5 \left(\ln\left(y + \sqrt{y^2 - 1}\right)\right)^2 \mathrm{d}y$ ***(M1)(A1)***

> **Note:** Award ***M1*** for attempting to use $V = \pi \int_c^d x^2 \mathrm{d}y$.

$= 37.1 \left(\text{units}^3\right)$ ***A1***

***[8 marks]***

***Total [22 marks]***